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On the Distribution of Radium in the Earth's Crust, and on the Earth's Internal Heat

R. J. Strutt

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at the Hurlers, Professors Lloyd Morgan and Morrow and Mr. Dymond at Stanton Drew, and Messrs. H. Bolitho, H. Thomas and Captain Henderson in south-west Cornwall.

To Lord Falmouth and Mr. Wallis I am also under obligations, as they were good enough to assist my inquiries by allowing an opening to be made in a stone wall at the Merry Maidens to view the alignment to the Pipers.

On the Distribution of Radium in the Earth's Crust, and on the Earth's Internal Heat.

By the Hon. R. J. STRUTT, F.R.S., Fellow of Trinity College, Cambridge.

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§ 1.—*Introduction.*

Professor Rutherford[*] has given a calculation which suggests that there may be enough radium in the earth to account for the temperature gradient observed near the surface.

The question is of great interest from a cosmical point of view. For if we find that the earth's internal heat is due to radio-activity, and if we assume, as has been usual, that this heat is due to some vestiges of the cause operative in the sun and stars, it would follow that these latter are heated by radio-active changes also.

Professor Rutherford's calculation was based on some data given by Elster and Geitel on the amount of radium emanation which diffused out from a sample of clay. These data were obtained at a time when the quantitative determination of minute amounts of radium was not well understood, and are moreover inadequate to give any general idea of the average amount

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[*] 'Radio-activity,' p. 494, 2nd Edition.

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of radium in the earth's crust. I have, therefore, made an extensive investigation of the amount of radium in various representative rocks. This forms the subject of the present paper. The results are very surprising. Considerable detail will therefore be given, in order to enable readers to judge whether there is any probability of these results being substantially incorrect.

§ 2.—*Choice of Material.*

The earth's crust, which is alone accessible to us, consists of igneous rocks, and of sedimentary material which results from the action of geological changes upon these rocks. No doubt the average radium content of the original igneous rocks might be inferred fairly well from the examination of a large number of sedimentary ones. It is, however, much more satisfactory to examine the igneous materials directly, for then they can be classed among themselves as to their radium content.

I have examined a few sedimentary rocks, but do not attach much importance to them, and rely chiefly on results obtained with original igneous material for determining the average radium content of the earth's crust. The results with regard to sedimentary rocks will be given in a future paper. I hope also to determine which of the numerous minerals contained in igneous rocks carry the radium, a subject not touched in the present communication.

Meteorites have a special interest of their own. A few determinations have been made on them, and are incidentally included in this paper.

§ 3.—*Method of Determining Radium Content.*

Radium in the rocks was quantitatively determined by means of its emanation. A solution of the rock was stored till the emanation had accumulated. This latter was then extracted by boiling, and introduced into an electroscope. The increased rate of leak produced was a measure of the amount of radium present. This measure was made absolute by going through the same process with a uranium mineral of known radium content.

In order to make certain of extracting the emanation quantitatively, it is essential to decompose the rock completely by chemical agency. In the case of limestones and metallic meteorites, this can be effected by solution in hydrochloric acid, but in the case of siliceous materials, fusion with alkaline carbonate is necessary.

The standard procedure was as follows:—Fifty grammes of the rock was broken up in an iron mortar, and then finely ground in an agate one, until it would pass through a sieve of 90 threads to the inch. This process could be carried out by an unskilled assistant in less than an hour, and ensured easy decomposition of the rock.

Two hundred and fifty grammes of a mixture of anhydrous sodium and potassium carbonates was melted in a large platinum basin. For this purpose the basin was surrounded with an extemporised furnace casing of asbestos millboard, and heated from below by means of a gas blowpipe. The blowpipe was supplied with air from an automatic blowing apparatus worked by water pressure. As soon as the carbonate was melted, the rock powder was thrown on to its surface in small portions at a time until it had all been added. The fusion was usually continued for about an hour after all effervescence was over.

The residue was then digested with hot water to dissolve out the alkaline silicates and carbonates. The portion insoluble in water was dissolved in hydrochloric acid. Some silica always separated from the acid solution, and was allowed to remain floating in it.

The two solutions, aqueous and acid, were set aside in separate flasks closed with indiarubber stoppers. Mixing them was avoided, because of the bulky and unmanageable precipitate of silica which would have been thrown down.

Decomposition with hydrofluoric acid has some advantages over the use of sodium carbonate. It requires, however, more minute pulverisation of the material, and is more costly in practice owing to difficulties of carriage and storage. After one or two trials its use was abandoned.

The two flasks containing the respective solutions were allowed to stand for some determinate number of days. A week was the minimum. More commonly a fortnight or three weeks was allowed. During this time, any radium contained in the rock was generating emanation. After three weeks the quantity has practically reached a maximum. The fraction of this maximum generated in any lesser period could be calculated from the equation for the rise of activity, originally given by Rutherford and Soddy.

I described a method for quantitatively extracting the emanation in 'Roy. Soc. Proc.,' A, vol. 76, 1905, p. 89. An improved modification of that method has been employed in the present investigation. The flask A (see figure) contains the solution. The accumulated emanation is at first partly dissolved, partly contained in the air which occupies the upper part of the flask. The flask is uncorked and attached to the lower end of the condenser B as rapidly as possible, so as to avoid loss of emanation. A is then boiled to expel radium emanation. The steam which issues is condensed in B and drops back. The air charged with emanation passes out

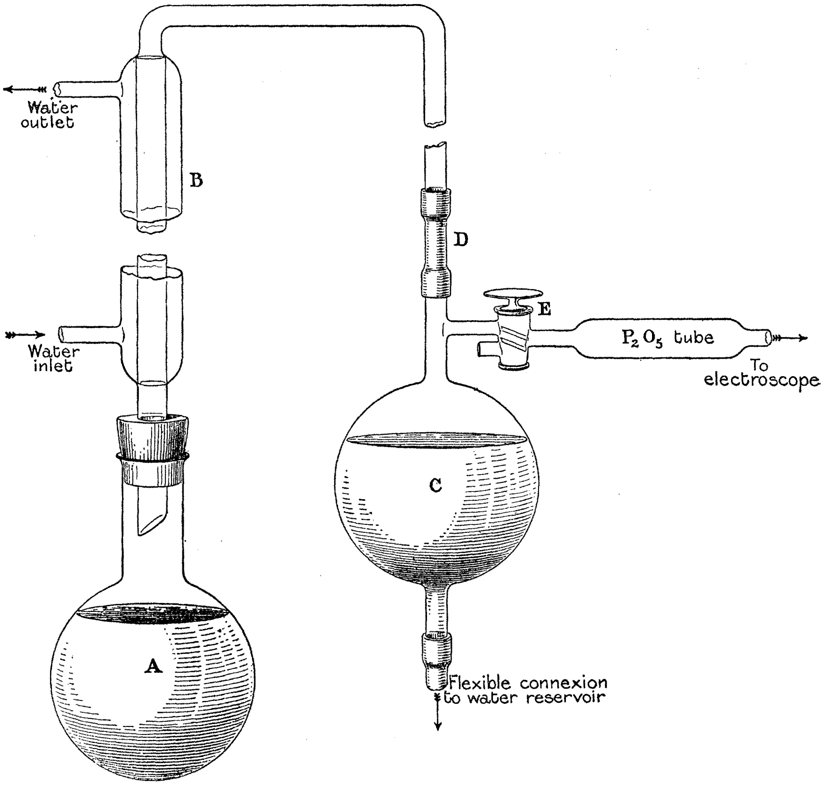

Fig. 1.

into the gasholder C, displacing the water which previously filled it. This boiling is continued for one hour. At the end of that time, the cooling water is run off from the jacket of B. Steam is allowed to pass so as to wash out all air and emanation from A and from the connecting tubes into C. The indiarubber connection at *d* is then nipped,* and the burner under the flask immediately withdrawn.

In this way the emanation is collected in the gasholder *c*. It merely remains to transfer it to the electroscope when cold. The latter is exhausted, and the emanation allowed to pass into it through the stop-cock E and a drying tube. Air is then admitted to the electroscope up to atmospheric pressure. After an interval of three hours, to allow the active deposit to form, the rate of leak is read.

The normal leak of the electroscope was repeatedly determined in the course of the investigation.

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* To do this conveniently, it is very desirable to have the pinchcock attached to a firm support, so that it can be screwed up with one hand.

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The following are the values in scale-divisions per hour, 24, 25·3, 24·1, 24, 23, 20·6, 21·7, 22·3, 22·5, 22·1. Mean, 23 nearly. These values were taken soon after exhaustion of the electroscope and admission of air.

If the instrument was left closed, the leak was found to have risen appreciably, after the lapse of a day. A similar effect has been noticed at the Cavendish Laboratory, and is, I believe, under investigation there. It does not come into question here, since time was never allowed for it to enter.

In work of this kind, when the effect to be looked for is small, it is most necessary to make certain that the emanation really comes from the material under investigation, and not from any extraneous source. I was deceived in this way in concluding that mercury gave off an emanation. In the present case, every precaution was taken.

The laboratory had never had radio-active materials introduced into it. Solutions of the reagents employed were separately tested for emanation, after they had stood closed for a fortnight, with the following results:—

	Rate of leak.
Sodium carbonate, 250 grammes	23·9
Potassium carbonate, 250 grammes	24·2
Hydrochloric acid, 500 c.c.	24·0
Water (Cambridge supply) 2000 c.c.	24·0

In none of these cases does the leak measurably exceed that normal to the electroscope. It may, therefore, be concluded that the reagents used to decompose the rocks are not responsible for the emanation obtained.

Cambridge tap water was used to make the solutions and to fill the gas-holder C (figure). This water originally contains dissolved emanation, as Professor J. J. Thomson has shown. It was therefore carefully boiled to expel this before use. The test given above was made on water which had already been boiled before setting it aside, and shows that no measurable quantity of emanation is generated by dissolved material when the original supply has been expelled. The (boiled) water in the gasholder was changed after each experiment.

Rutherford and Boltwood* have determined the radium content of uranium minerals in absolute measure. They find that the radium associated with 1 gramme of uranium is $7{\cdot}4 \times 10^{-7}$ gramme. I use this value rather than my own† because they tested the heat production of their standard radium, while I had no test for the purity of mine.

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* 'Amer. Journ. Sci., 1905, p. 55.

† 'Roy. Soc. Proc.,' A, vol. 76, 1905, p. 88.

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The uranium minerals used for standardisation were Torbernite (copper uranium phosphate) containing 60 per cent. U_3O_8 (= 51 per cent. uranium) and pitchblende, containing 73·5 per cent. U_3O_8 (= 62·4 per cent. uranium).

The rate of leak due to the emanation produced by a few milligrammes of each of these in *one day* was determined. From this the quantity of radium was deduced, which would in an *indefinite time* produce enough emanation to give a leak of one division per hour. This is the constant given in the last column of the following table.

Standardisation Experiments.

Mineral.	Percentage of uranium (metal).	Quantity, grammes.	Rate of leak* due to emanation accumulated in one day.	Constant deduced.
Torbernite	51·0	0·0076	214·0	$2{\cdot}3 \times 10^{-12}$
" same sample......	51·0	0·0076	209·0	$2{\cdot}36 \times 10^{-12}$
Torbernite	51·0	0·0046	116·5	$2{\cdot}56 \times 10^{-12}$
" same sample......	51·0	0·0046	113·0	$2{\cdot}64 \times 10^{-12}$
Pitchblende	62·4	0·0063	207·0	$2{\cdot}41 \times 10^{-12}$
" same sample ...	62·4	0·0063	202·0	$2{\cdot}45 \times 10^{-12}$

* Corrected for normal leak of the electroscope.

The mean value for the constant is $2{\cdot}45 \times 10^{-12}$, or in other words a leak equal to one scale-division per hour represents $2{\cdot}45 \times 10^{-12}$ gramme of radium in the sample examined, if the latter has had time to produce its maximum amount of emanation.

Two specimen determinations will now be given in detail, the first of granite from the Cape of Good Hope, the second of olivine rock from the Isle of Rum. These are respectively representative of high and low radium content.

Cape Granite, 50 grammes.—Solutions stood from March 21 to March 26, giving 60 per cent. of the maximum amount of emanation.

	Scale-div. per hour.	Corrected.
Emanation from acid solution	103	80·0
Emanation from alkaline solution ...	31	8·0

Total ...... 88·0 scale-div. per hour.

Thus the equilibrium amount of emanation would give 146 scale-divisions per hour, or per gramme of rock $\frac{146}{50} = 2{\cdot}92$ scale-divisions per hour. Thus 1 gramme of rock contains $2{\cdot}92 \times 2{\cdot}45 \times 10^{-12}$ gramme or $7{\cdot}15 \times 10^{-12}$ gramme radium.

Olivine Rock, Isle of Rum, 50 grammes.—Solutions stood from January 31 to February 19, giving practically the equilibrium amount of emanation.

	Scale-div. per hour.	Corrected.
Emanation from acid solution ……	34·0	11·0
Emanation from alkaline solution…	25·8	2·8

Total …… 13·8 scale-div. per hour.

One gramme of rock would give 0·276 scale-division per hour, and contains $2{\cdot}45 \times 0{\cdot}276 \times 10^{-12} = 0{\cdot}676 \times 10^{-12}$ gramme radium.

This last example represents, as mentioned above, nearly the lowest radium content encountered among igneous rocks. It will be noticed that the leak produced by the emanation is, even in this unfavourable case, about half that normal to the electroscope, and is, therefore, quite well marked. It will be noticed also that the alkaline solution contains only a small proportion of the total radium present in the original rock.

§ 4.—*Results for Igneous Rocks.*

The results for igneous rocks will now be given in tabular form and in order of radium content (see next page).

It will be observed that, in general, rocks like granite, with a high percentage of silica, are richer in radium than basic rocks. The rule, however, is by no means invariable.

Uranium ores occur in several of the rocks examined; thus the granites Nos. 2 and 6 occur near the pitchblende bearing veins which were worked at Wheal Trenwith, St. Ives. The Norwegian syenite rocks, Nos. 3, 8, and 15, also contain local deposits of uranium bearing minerals. These various rocks are fairly rich in radium, but do not stand in a class by themselves.

Thus the concentration of radium in such deposits is extremely local, and cannot appreciably disturb any general conclusions as to the total amount of radium in the earth's crust.

Confirmatory of this conclusion is the fact that the temperature gradient at Wheal Trenwith, St. Ives, is quite normal.* Considerable quantities of pitchblende occur in this mine, but they are evidently insufficient to cause appreciable disturbance in the distribution of temperature.

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* See Prestwich, 'Controverted Questions in Geology, p. 216.

No.	Name of rock.	Locality.	Density.	Radium per gramme, in grammes.	Radium per c.c., in grammes.
1	Granite	Rhodesia	2·63	$9{\cdot}56 \times 10^{-12}$	$25{\cdot}2 \times 10^{-12}$
2	„	Lamorna Quarry, Cornwall	2·62	$9{\cdot}35 \times 10^{-12}$	$24{\cdot}5 \times 10^{-12}$
3	Zircon syenite	Brevig, Norway	2·74	$9{\cdot}30 \times 10^{-12}$	$25{\cdot}5 \times 10^{-12}$
4	Granite	Rosemorran, Cornwall (?)	2·62	$8{\cdot}43 \times 10^{-12}$	$22{\cdot}1 \times 10^{-12}$
5	„	Cape of Good Hope	2·67	$7{\cdot}15 \times 10^{-12}$	$19{\cdot}1 \times 10^{-12}$
6	„	Knill's Monument, near Carbis Bay, St. Ives, Cornwall	2·61	$6{\cdot}90 \times 10^{-12}$	$18{\cdot}0 \times 10^{-12}$
7	„	Shap Fell, Westmoreland	2·65	$6{\cdot}63 \times 10^{-12}$	$17{\cdot}6 \times 10^{-12}$
8	Elæolite syenite	Laurdal, Norway	2·70	$4{\cdot}88 \times 10^{-12}$	$13{\cdot}2 \times 10^{-12}$
9	Granite	Haytor, Devonshire ?	2·61	$3{\cdot}69 \times 10^{-12}$	$9{\cdot}64 \times 10^{-12}$
10	Blue ground	Kimberley	3·06	$3{\cdot}37 \times 10^{-12}$	$10{\cdot}3 \times 10^{-12}$
11	Leucite basanite	Mt. Somma, Vesuvius	2·72	$3{\cdot}33 \times 10^{-12}$	$9{\cdot}07 \times 10^{-12}$
12	Hornblende granite	Assouan, Egypt	2·64	$2{\cdot}45 \times 10^{-12}$	$6{\cdot}47 \times 10^{-12}$
13	Pitchstone	Isle of Eigg	2·41	$2{\cdot}06 \times 10^{-12}$	$4{\cdot}97 \times 10^{-12}$
14	Hornblende diorite	Schriesheim, near Heidelberg	2·89	$1{\cdot}98 \times 10^{-12}$	$5{\cdot}73 \times 10^{-12}$
15	Augite syenite	Laurvig, Norway	2·73	$1{\cdot}86 \times 10^{-12}$	$5{\cdot}07 \times 10^{-12}$
16	Peridotite	Isle of Rum	3·15	$1{\cdot}37 \times 10^{-12}$	$4{\cdot}32 \times 10^{-12}$
17	Olivine basalt	Talisker Bay, Skye	2·89	$1{\cdot}32 \times 10^{-12}$	$3{\cdot}82 \times 10^{-12}$
18	Olivine euchrite	Isle of Rum	2·97	$1{\cdot}28 \times 10^{-12}$	$3{\cdot}80 \times 10^{-12}$
19	Basalt	Victoria Falls	2·75	$1{\cdot}26 \times 10^{-12}$	$3{\cdot}46 \times 10^{-12}$
20	Hornblende granite	Mt. Sorrel, Leicestershire	2·71	$1{\cdot}25 \times 10^{-12}$	$3{\cdot}38 \times 10^{-12}$
21	Dolerite	Isle of Canna	2·95	$1{\cdot}24 \times 10^{-12}$	$3{\cdot}65 \times 10^{-12}$
22	Greenstone	Carrick Dû, St. Ives	2·99	$1{\cdot}14 \times 10^{-12}$	$3{\cdot}41 \times 10^{-12}$
23	Basalt	Giants' Causeway, Antrim	2·80	$1{\cdot}03 \times 10^{-12}$	$2{\cdot}89 \times 10^{-12}$
24	Serpentine	Cadgwith, Lizard	2·60	$1{\cdot}00 \times 10^{-12}$	$2{\cdot}60 \times 10^{-12}$
25	Granite	Isle of Rum	2·61	$0{\cdot}723 \times 10^{-12}$	$1{\cdot}89 \times 10^{-12}$
26	Olivine rock	Isle of Rum	3·22	$0{\cdot}676 \times 10^{-12}$	$2{\cdot}18 \times 10^{-12}$
27	Dunite	L. Scaivig	3·34	$0{\cdot}664 \times 10^{-12}$	$2{\cdot}22 \times 10^{-12}$
28	Basalt	Ovifak, Disco Island, Greenland	3·01	$0{\cdot}613 \times 10^{-12}$	$1{\cdot}84 \times 10^{-12}$

§ 5.—*Meteorites.*

Determinations have been made on one stony meteorite, on three samples of meteoric iron, and on a sample of native iron from Ovifak, Disco Island, Greenland. The quantities available have been various, and are entered on the list of results. Where the radium is entered as 0, it is to be understood that no leak was obtained which could clearly be distinguished from that normal to the electroscope.

It will be observed that the stony meteorite contains about as much radium as those basic terrestrial rocks, which it resembles in general composition. No evidence was obtained of the presence of radium in iron meteorites. The Greenland iron contains a little radium. This was probably present in the siliceous material contained in this iron, which had been filtered off, decomposed by fusion and added to the main solution.

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Material.	Locality.	Quantity taken.	Radium per gramme.
Stony meteorite	Dhurmsala	50 grammes	$1{\cdot}12 \times 10^{-12}$
Meteoric iron	Thunda	60 ,,	0
,, ,, ...	Augusta Co., Virginia	32 ,,	0
,, ,, ...	Santa Catarina	50 ,,	0
Native iron..................	Disco Island, Greenland	200 ,,	$0{\cdot}424 \times 10^{-12}$

§ 6.—*The Earth's Internal Heat.*

If q be the mean mass of radium per cubic centimetre in the earth, h the heat production of radium per gramme per second, then the total heat production must be, per second,

$$q \, . \, h \, . \, \tfrac{4}{3}\pi \mathrm{R}^3.$$

If k be the thermal conductivity of the surface rocks, then the total outflow of heat per second will be

$$4\pi \mathrm{R}^2 k \, . \left(\frac{d\theta}{dr}\right)_{\mathrm{R}},$$

where $\left(\frac{d\theta}{dr}\right)_{\mathrm{R}}$ is the temperature gradient near the surface, as observed experimentally.

If the earth is in a thermally steady state, then these expressions will be equal, *i.e.*,

$$4\pi \mathrm{R}^2 k \left(\frac{d\theta}{dr}\right)_{\mathrm{R}} = \tfrac{4}{3}\pi \mathrm{R}^3 q h,$$

or

$$q = \frac{3k}{h\mathrm{R}} \left(\frac{d\theta}{dr}\right)_{\mathrm{R}}.$$

For k I take the value 0·0041.* For the temperature gradient 1° F. in 42·4† feet, or, in C.G.S. system, $4{\cdot}3 \times 10^{-4}$.

One gramme of radium *bromide* produces 100 calories per hour. This gives for h the value $4{\cdot}75 \times 10^{-2}$.

R, the earth's radius, $= 6{\cdot}38 \times 10^8$.

Thus

$$q = \frac{3 \times 4{\cdot}1 \times 10^{-3} \times 4{\cdot}3 \times 10^{-4}}{4{\cdot}75 \times 10^{-2} \times 6{\cdot}38 \times 10^8} = 1{\cdot}75 \times 10^{-13}.$$

If, therefore, we assume the earth to be in thermal equilibrium, then, even if the whole of the internal heat is due to radium, the mean quantity per cubic centimetre cannot much exceed $1{\cdot}75 \times 10^{-13}$ gramme per cubic

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* See Prestwich, *loc. cit.*

† *Loc. cit.*

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centimetre, always supposing that the heat production of radium is not materially diminished under the conditions prevailing inside the earth.

Rutherford,[*] taking somewhat different values for the constants involved, gave the value $2 \cdot 6 \times 10^{-13}$ *radium bromide*, equivalent to $1 \cdot 52 \times 10^{-13}$ gramme *radium* per cubic centimetre.

It will be observed that all the igneous rocks examined, without exception, contain far more radium per cubic centimetre than this. The poorest of all, Greenland basalt, contains more than 10 times as much; an average rock something like 50 or 60 times.

The question must be faced: Why has not the earth a temperature gradient far larger than that observed?

The calculation given above assumes,—

(1) That the earth is in thermal equilibrium, *i.e.*, that the amount of heat which escapes per second is equal to the supply produced in that time.

(2) That no other source of internal heat than radium exists.

(3) That a gramme of radium produces as much heat inside the earth as at the surface.

As to the first assumption, to suppose that the earth is cooling only aggravates the difficulty. To assume that it is getting hotter is an explanation not likely to be regarded with favour. I have not yet considered it quantitatively.

As to the second assumption, there can be little doubt that a quantity of uranium proportional to the amount of radium exists in the rocks.[†] Moreover, a trace of thorium is not improbably present. These sources of heat are not likely to be important, as compared with radium. There is, too, the possibility of the radio-activity of ordinary materials. If, however, a heating effect from them is assumed, of the order of magnitude to be expected from the ionisation they give, a temperature gradient would result something like 1000 times larger than that observed. I think this argument is conclusive against the theory that they have a genuine radio-activity of this order of magnitude.[‡]

There remains the third assumption, stated above. This cannot be passed over so lightly as the others, but will be more conveniently discussed later. I shall suppose for the moment that it is justified, and that the earth cannot contain more on the average than $1 \cdot 75 \times 10^{-13}$ gramme of radium per cubic centimetre.

For surface rocks the experiments show that 5×10^{-12} gramme per cubic

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[*] *Loc. cit.*

[†] Too little to be detected in the ordinary course of analysis.

[‡] See 'Nature,' December 21, 1905.

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centimetre is a representative value. This is, if anything, an understatement. Thus not more than about 1/30 of the earth's volume can consist of material similar to that encountered on the surface. This gives a depth of about 45 miles for the rocky crust, assuming the total absence of radio-active material within.

I shall next consider the distribution of temperature in a crust of this thickness. The curvature of so thin a crust is relatively small, and may, without appreciable sacrifice of accuracy, be disregarded. Let x be the depth at any point, measured from the outer surface, q the quantity of radium per cubic centimetre, h the heat developed in one second by 1 gramme of radium, θ the temperature at the depth x, and k the thermal conductivity.

Then the equation of conduction is

$$\frac{d^2\theta}{dx^2} = \frac{-qh}{k},$$

which gives by integration

$$\theta = \frac{-qh}{2k}(x^2+ax+b),$$

where a and b are constants.

If the surface of the earth be assumed to be at 0° C., then $\theta = 0$ when $x = 0$.

If t be the thickness of the crust, $d\theta/dx = 0$, when $x = t$. This is clear, since the temperature must be a maximum at the inner boundary of the crust. The interior core, free from radium, must be at a uniform temperature throughout.

Substituting these values we get

$$b = 0 \quad \text{and} \quad a = -2t.$$

Hence

$$\theta = \frac{qhx}{2k}(2t-x).$$

Substituting the numerical values adopted in this paper,

$$\theta = \frac{5\times10^{-12}\times4{\cdot}75\times10^{-2}x\,(2\times7{\cdot}25\times10^{6}-x)}{2\times4{\cdot}1\times10^{-3}} = 2{\cdot}90\times10^{-11}x\,(1{\cdot}45\times10^{7}-x).$$

The curve appended shows this distribution of temperature graphically.

The maximum temperature at the bottom of the crust will be where $x = 7{\cdot}25\times10^{6}$. This gives $\theta = 2{\cdot}90\times10^{-11}\times(7{\cdot}25)^2\times10^{12} = 1530°$, a temperature considerably below the melting point of platinum.*

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* No doubt the assumption of constant conductivity at all temperatures is an unsatisfactory feature in this calculation. It is difficult to know what other assumption to make, however. Some experiments of Lord Kelvin and Mr. Erskine Murray ('Roy. Soc. Proc.,' vol. 58, p. 162) seem to indicate a diminution of conductivity with temperature.

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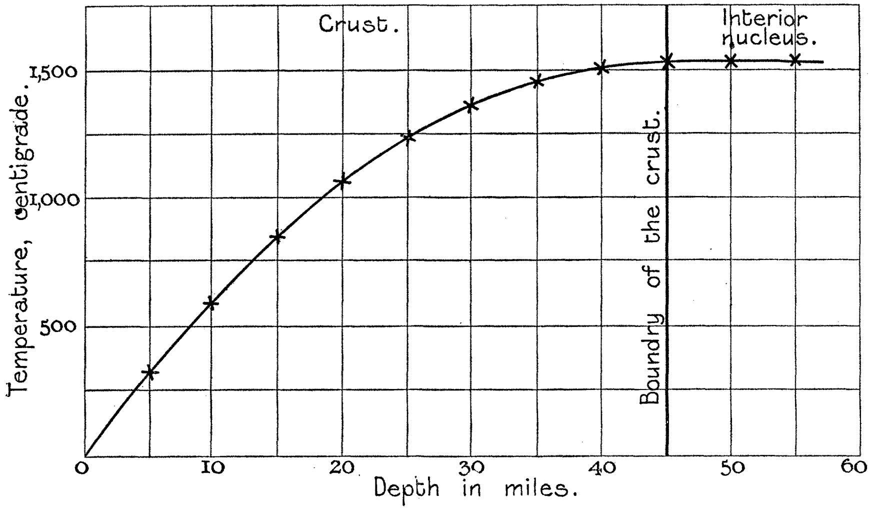

Fig. 2.—Calculated distribution of temperature in the earth's interior.

This result has been obtained on the provisional assumption that the heat production of radium is the same throughout the earth's crust as under surface conditions. In justification of this, a paper by Mr. W. Makower* may be referred to. The activity of radium emanation and its products are shown to be substantially the same at temperatures of 1200° as at the ordinary temperature, though evidence was obtained of a slight change of activity in one of the products. Thus there is no reason at present to think that notable change of activity sets in before 1500° is reached. I wish to express myself with some reserve on this subject. Further experiments might conceivably show a rapid loss of activity as this temperature was approached. In that case the conclusions here drawn as to the earth's internal condition would require modification.

I was inclined at first to think it incredible that the earth's crust could have so small a thickness as 45 miles, and was therefore much interested to hear that Professor Milne† had come to a substantially identical conclusion, from a study of the velocity of propagation of earthquakes through the earth's interior. He gives 30 miles as the thickness. This is quite

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On the other hand, Mr. C. H. Lees found the thermal conductivity of window glass to increase with temperature; and at high temperatures rock magmas must approach the quality of glass; indeed, they sometimes even retain that quality on cooling (obsidian and pitchstone). The mean thermal conductivity of rock cannot be much *more* than that assumed, for otherwise the internal temperature would not be high enough to produce volcanic phenomena.

[*] 'Roy. Soc. Proc.,' A, vol. 77, p. 241.

[†] Bakerian Lecture, 1906.

consistent with my data, if rocks like granite, rich in radium, are assumed to be somewhat more abundant than I have supposed, in taking the value 5×10^{-12} gramme radium per cubic centimetre as representative.

Professor Milne expresses the opinion that a fairly abrupt transition occurs at a depth of 30 miles, and that the material below that depth is fairly uniform throughout the globe. This is entirely in agreement with the view put forward above with regard to the earth's interior.

The chemical nature of the interior is a difficult problem. It can scarcely consist mainly of iron, as has been very commonly supposed, from the analogy of meteorites. Meteoric iron is remarkably free from radium, as shown above, and in this respect answers the requirements well enough. But if the stony exterior of the earth is but a small fraction of the whole volume, it cannot have much influence on the mean density, which should be nearly equal to that of the core. The density of the earth (5·5) is much less than the density of iron (7·7).

§ 7.—*Internal Heat of the Moon.*

The data of this paper have an interesting application to the moon. What we can observe of the moon's surface suggests that it consists of rock like that on the earth. The moon is believed to have originally separated from the earth's surface, and should, therefore, consist of the same material as the latter. Moreover, the density of the moon (3·5) does not differ much from that of rock. It seems reasonable to conclude from these facts that the moon consists almost entirely of rock similar to that of the earth's crust.

On this view, the temperature gradient of the moon should be very great in comparison with that of the earth. The material of the moon is taken to be some 30 times richer in radium than the (mean) material of the earth. Her volume is about one-fiftieth that of the earth. Thus the total heat production in the moon would be about half of that in the earth. This heat has to flow out through about one-sixteenth the area of the earth surface. Thus the temperature gradient at the moon's surface should be eight times greater than at the earth's.

In addition to this, gravity is very much less on the moon. We may conclude that the conditions which prevail there are far more favourable to manifestation of the internal heat by volcanic upheaval. This fully explains why volcanic features are so much more prominent on the moon than on the earth.

It has generally been supposed that the lunar craters are extinct. But that view seems to rest chiefly on an *a priori* conviction that the moon has no internal heat. As Professor W. H. Pickering has pointed out, all those observers who have made a special study of the moon have believed in the reality of changes occurring there.*

§ 8.—*Summary of Conclusions.*

1. Radium can easily be detected in all igneous rocks. Granites, as a rule, contain most radium, basic rocks the least.

2. This distribution of radium is uniform enough to enable a fair estimate to be made of the total quantity in each mile of depth of the crust.

3. The result indicates that the crust cannot be much more than 45 miles deep, for otherwise the outflow of heat would be greater than is observed to be the case. The interior must consist of some totally different material. This agrees entirely with Professor Milne's conclusion drawn from a study of the velocity of propagation of earthquake shocks through the interior.

4. The moon probably consists for the most part of rock, and if so, its internal temperature must be far greater than that of the earth. This explains the great development of volcanoes on the moon.

5. Iron meteorites contain little, if any, radium. Stony ones contain about as much as the terrestrial rocks which they resemble.

In conclusion, I must thank Mr. A. Harker, Mr. Clement Reid, and Mr. A. Hutchinson for their kindness in providing me with various specimens of rock, and for information on geological matters.

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* 'Nature,' January 5, 1905.

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